

ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ ΚΑΙ ΠΟΛΙΤΙΣΜΟΥ
ΔΙΕΥΘΥΝΣΗ ΑΝΩΤΕΡΗΣ ΚΑΙ ΑΝΩΤΑΤΗΣ ΕΚΠΑΙΔΕΥΣΗΣ
ΥΠΗΡΕΣΙΑ ΕΞΕΤΑΣΕΩΝ

ΠΑΓΚΥΠΡΙΕΣ ΕΞΕΤΑΣΕΙΣ 2006

Μάθημα : ΜΑΘΗΜΑΤΙΚΑ ΠΡΑΚΤΙΚΗΣ ΚΑΤΕΥΘΥΝΣΗΣ
4-ΩΡΟ ΤΕΧΝΙΚΩΝ ΣΧΟΛΩΝ

Ημερομηνία και ώρα εξέτασης: Δευτέρα, 29 Μαΐου 2006
11.00 – 14.00

ΛΥΣΕΙΣ

ΜΕΡΟΣ Α

1.	$V = \alpha \cdot \beta \cdot \gamma$ $V = 7 \cdot 4 \cdot 5$ $V = 140 \text{ cm}^3$	
2.	$\frac{75}{100} \cdot 188 = 141$ Θα πληρώσει £141	
3.	$\int (5x + 2) dx = \frac{5x^2}{2} + 2x + c$	
4.	$(x - \alpha)^2 + (y - \beta)^2 = R^2$ $(x - 2)^2 + (y + 3)^2 = 5^2$ $(x - 2)^2 + (y + 3)^2 = 25$	
5.	α) Θα πληρώσει για ενοίκιο £200. β) Ξοδεύει συνολικά κάθε μήνα $200 + 150 + 250 + 100 = £700$. γ) Του περισσεύουν κάθε μήνα $1000 - 700 = £300$. $\frac{300}{1000} \cdot 100 = 30\%$ των εσόδων του.	
6.	$M_8^e = \frac{8!}{2!} = 20160$ $M_6 = 6! = 720$	

7.	<p>α) $\frac{dy}{dx} = 12x^3 + 5$</p> <p>β) $\frac{dy}{dx} = \frac{3x^2\eta\mu x - x^3\sigma\upsilon\nu x}{\eta\mu^2 x}$</p>	
8.	<p>$P(A) = \frac{3}{6} = \frac{1}{2}$</p> <p>$P(B) = \frac{2}{6} = \frac{1}{3}$</p> <p>$P(\Gamma) = \frac{5}{6}$</p>	
9.	<p>$\left. \begin{array}{l} 3x + y = 8 \\ x \cdot y = -3 \end{array} \right\} \Rightarrow \begin{array}{l} y = 8 - 3x \\ x(8 - 3x) = -3 \end{array}$</p> <p>$8x - 3x^2 + 3 = 0$</p> <p>$3x^2 - 8x - 3 = 0$</p> <p>$x_{1,2} = \frac{8 \pm \sqrt{64 - 4 \cdot 3(-3)}}{2 \cdot 3} = \frac{8 \pm \sqrt{100}}{6} = \frac{8 \pm 10}{6}$</p> <p>$x_1 = \frac{18}{6} = 3, \quad \Rightarrow y_1 = 8 - 3 \cdot 3 = -1 \quad (3, -1)$</p> <p>$x_2 = \frac{-2}{6} = -\frac{1}{3}, \quad \Rightarrow y_2 = 8 - 3(-\frac{1}{3}) = 9 \quad (-\frac{1}{3}, 9)$</p>	
10.	<p>$2\sigma\upsilon\nu^2 x - 5\sigma\upsilon\nu x + 2 = 0 \quad \Theta\acute{\epsilon}\tau\omega \sigma\upsilon\nu x = \omega$</p> <p>$2\omega^2 - 5\omega + 2 = 0$</p> <p>$\omega_{1,2} = \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm 3}{4} = \begin{cases} \omega_1 = 2 \\ \omega_2 = \frac{1}{2} \end{cases}$</p> <p>$\sigma\upsilon\nu x = 2 \quad \text{απορρίπτεται}$</p> <p>$\sigma\upsilon\nu x = \frac{1}{2} \Rightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu 60^\circ$</p> <p>Γενικές λύσεις : $x = 360^\circ \kappa \pm 60^\circ, \kappa \in \mathbb{Z}$</p>	

ΜΕΡΟΣ Β΄

<p>B1</p>	<p>Πλήρωσε για ανακαίνιση $\frac{65}{100} \cdot 40\ 000 = \text{£}26\ 000$</p> <p>Πλήρωσε συνολικά $90\ 000 + 26\ 000 + 8\ 000 + 3\ 500 = \text{£}127\ 500$</p> <p>Το σπίτι πρέπει να πωληθεί</p> <p>$\frac{120}{100} \cdot 127\ 500 = \text{£}153\ 000$</p>																																																																			
<p>B2</p>	<p>$P(A) = \frac{\binom{7}{3}}{\binom{12}{3}} = \frac{35}{220} = \frac{7}{44}$</p> <p>$P(B) = \frac{\binom{7}{2}\binom{5}{1}}{\binom{12}{3}} = \frac{21 \cdot 5}{220} = \frac{21}{44}$</p> <p>$P(\Gamma) = \frac{\binom{7}{3} + \binom{7}{2}\binom{5}{1}}{\binom{12}{3}} = \frac{35 + 105}{220} = \frac{140}{220} = \frac{7}{11}$</p> <p>ή</p> <p>$P(\Gamma) = P(A) + P(B) = \frac{7}{44} + \frac{21}{44} = \frac{28}{44} = \frac{7}{11}$</p>																																																																			
<p>B3</p>	<p>α)</p> <table border="1" data-bbox="312 1480 1195 2085"> <thead> <tr> <th>Ώρες x_i</th> <th>παιδιά f_i</th> <th>$f_i \cdot x_i$</th> <th>$x_i - \bar{x}$</th> <th>$(x_i - \bar{x})^2$</th> <th>$f_i(x_i - \bar{x})^2$</th> </tr> </thead> <tbody> <tr><td>0</td><td>3</td><td>0</td><td>-3</td><td>9</td><td>27</td></tr> <tr><td>1</td><td>8</td><td>8</td><td>-2</td><td>4</td><td>32</td></tr> <tr><td>2</td><td>10</td><td>20</td><td>-1</td><td>1</td><td>10</td></tr> <tr><td>3</td><td>14</td><td>42</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>4</td><td>6</td><td>24</td><td>1</td><td>1</td><td>6</td></tr> <tr><td>5</td><td>2</td><td>10</td><td>2</td><td>4</td><td>8</td></tr> <tr><td>6</td><td>4</td><td>24</td><td>3</td><td>9</td><td>36</td></tr> <tr><td>7</td><td>2</td><td>14</td><td>4</td><td>16</td><td>32</td></tr> <tr><td>8</td><td>1</td><td>8</td><td>5</td><td>25</td><td>25</td></tr> <tr> <td></td> <td>50</td> <td>150</td> <td></td> <td></td> <td>176</td> </tr> </tbody> </table>	Ώρες x_i	παιδιά f_i	$f_i \cdot x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$	0	3	0	-3	9	27	1	8	8	-2	4	32	2	10	20	-1	1	10	3	14	42	0	0	0	4	6	24	1	1	6	5	2	10	2	4	8	6	4	24	3	9	36	7	2	14	4	16	32	8	1	8	5	25	25		50	150			176	
Ώρες x_i	παιδιά f_i	$f_i \cdot x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$																																																															
0	3	0	-3	9	27																																																															
1	8	8	-2	4	32																																																															
2	10	20	-1	1	10																																																															
3	14	42	0	0	0																																																															
4	6	24	1	1	6																																																															
5	2	10	2	4	8																																																															
6	4	24	3	9	36																																																															
7	2	14	4	16	32																																																															
8	1	8	5	25	25																																																															
	50	150			176																																																															

$$(\alpha) \quad x_e = 3$$

$$\beta) \quad \bar{x} = \frac{\sum f_i \cdot x_i}{\sum f_i} = \frac{150}{50} = 3$$

$$\gamma) \quad \sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{v}} = \sqrt{\frac{176}{50}} = \sqrt{3,52} = 1,88$$

B4

$$\int (x^4 + 2\sqrt{x} + \frac{2}{x^3}) dx =$$

$$= \int (x^4 + 2x^{\frac{1}{2}} + 2x^{-3}) dx$$

$$= \frac{x^5}{5} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2 \frac{x^{-2}}{-2} + c$$

$$= \frac{x^5}{5} + \frac{4}{3} x^{\frac{3}{2}} - \frac{1}{x^2} + c$$

$$= \frac{x^5}{5} + \frac{4}{3} x\sqrt{x} - \frac{1}{x^2} + c$$

$$(\beta) \quad y = \eta\mu^2 x$$

$$\frac{dy}{dx} = 2\eta\mu x \cdot \sigma \sin x$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= 2\sigma \sin x \cdot \sigma \sin x + 2\eta\mu x (-\eta\mu x) \\ &= 2\sigma \sin^2 x - 2\eta\mu^2 x \end{aligned}$$

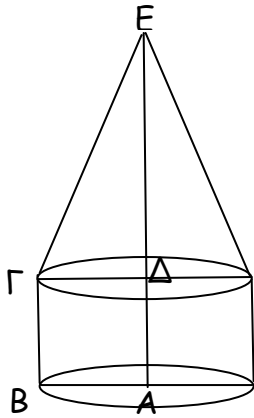
$$\frac{d^2y}{dx^2} + 4y = 2\sigma \sin^2 x - 2\eta\mu^2 x + 4\eta\mu^2 x$$

$$= 2\sigma \sin^2 x + 2\eta\mu^2 x$$

$$= 2(\sigma \sin^2 x + \eta\mu^2 x)$$

$$= 2 \cdot 1$$

$$= 2$$

B5

$$(\Delta E)^2 + (\Gamma \Delta)^2 = (\Gamma E)^2$$

$$(\Delta E)^2 + 5^2 = 13^2$$

$$(\Delta E)^2 + 25 = 169$$

$$(\Delta E)^2 = 169 - 25$$

$$(\Delta E)^2 = 144$$

$$\Delta E = 12 \text{ cm}$$

$$\begin{aligned} E_{\text{ολ}} &= E_{\text{B}} + E_{\text{κ. κυλίνδρου}} + E_{\text{κ. κώνου}} \\ &= \pi R^2 + 2\pi R \cdot u_{\text{κυλ}} + \pi R \cdot \lambda \\ &= \pi \cdot 5^2 + 2\pi \cdot 5 \cdot 5 + \pi \cdot 5 \cdot 13 \\ &= 25\pi + 50\pi + 65\pi \\ &= 25\pi + 50\pi + 65\pi \end{aligned}$$

$$E_{\text{ολ}} = 140\pi \text{ cm}^2$$

$$V_{\text{ολ}} = V_{\text{κυλίνδρου}} + V_{\text{κώνου}}$$

$$V_{\text{ολ}} = \pi R^2 \cdot u_{\text{κυλ}} + \frac{\pi R^2 \cdot u_{\text{κών}}}{3}$$

$$= \pi \cdot 5^2 \cdot 5 + \frac{\pi \cdot 5^2 \cdot 12}{3}$$

$$= 125\pi + 100\pi$$

$$V_{\text{ολ}} = 225\pi \text{ cm}^3$$