



2020 | Μάιος | Φάση 3 | Διαγωνίσματα Επανάληψης

ΑΛΓΕΒΡΑ

Β' Γενικού Λυκείου
Γενικής Παιδείας

Δευτέρα 25 Μαΐου 2020 | Διάρκεια Εξέτασης: 3 ώρες

ΑΠΑΝΤΗΣΕΙΣ

ΘΕΜΑ Α

- A1. Σχολικό βιβλίο σελ. 134.
- A2. Σχολικό βιβλίο σελ. 35.
- A3. i. → Σωστό, ii. → Λάθος, iii. → Σωστό, iv. → Λάθος, v. → Λάθος
- A4. i. $\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta - \eta\mu\alpha \cdot \eta\mu\beta$
ii. $\eta\mu(\alpha - \beta) = \eta\mu\alpha \cdot \cos\beta - \sin\alpha \cdot \eta\mu\beta$
iii. $\epsilon\varphi(\alpha - \beta) = \frac{\epsilon\varphi\alpha - \epsilon\varphi\beta}{1 + \epsilon\varphi\alpha \cdot \epsilon\varphi\beta}$
iv. $\epsilon\varphi 2\alpha = \frac{2\epsilon\varphi\alpha}{1 - \epsilon\varphi^2\alpha}$
v. $\sin^2\alpha - \eta\mu^2\alpha = \sin 2\alpha$.

ΘΕΜΑ Β

- B1. • $\eta\mu\left(\frac{9\pi}{2} - x\right) = \eta\mu\left(4\pi + \frac{\pi}{2} - x\right) = \eta\mu\left(\frac{\pi}{2} - x\right) = \sin x$.
• $\epsilon\varphi(5\pi - x) = \epsilon\varphi(4\pi + \pi - x) = \epsilon\varphi(\pi - x) = -\epsilon\varphi x$.
• $\sigma\varphi(21\pi + x) = \sigma\varphi(20\pi + \pi + x) = \sigma\varphi(\pi + x) = \sigma\varphi x$.



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- $\eta\mu(-x) = -\eta\mu x.$
- $\sigma\varphi\left(\frac{13\pi}{2} - x\right) = \sigma\varphi\left(6\pi + \frac{\pi}{2} - x\right) = \sigma\varphi\left(\frac{\pi}{2} - x\right) = \epsilon\varphi x.$
- $\sigma\varphi(-x) = -\sigma\varphi x.$
- $\sigma\upsilon\nu(3\pi - x) = \sigma\upsilon\nu(2\pi + \pi - x) = \sigma\upsilon\nu(\pi - x) = -\sigma\upsilon\nu x.$
- $\sigma\upsilon\nu\left(\frac{\pi}{2} - x\right) = \eta\mu x.$

$$\omega = \frac{\sigma\upsilon\nu x \cdot (-\epsilon\varphi x) \cdot \sigma\varphi x \cdot (-\eta\mu x)}{\epsilon\varphi x \cdot (-\sigma\varphi x) \cdot (-\sigma\upsilon\nu x) \cdot \eta\mu x} = 1.$$

B2. $f(x) = 3\eta\mu(2x)$

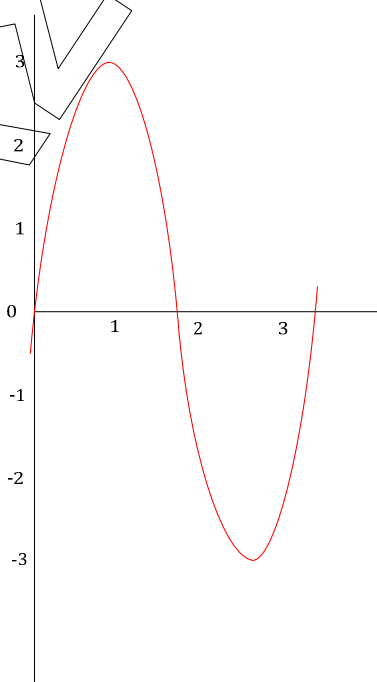
$$\omega = 2 \Leftrightarrow \frac{2\pi}{T} = 2 \Leftrightarrow T = \pi \text{ rad}$$

i.

$$-1 \leq \eta\mu(2x) \leq 1 \Leftrightarrow -3 \leq 3\eta\mu(2x) \leq 3.$$

Επομένως, $f_{\max} = 3,$ $f_{\min} = -3$

ii.





B3.

$$\eta\mu x \cdot \sigma\upsilon\nu x = \frac{\sqrt{18}}{12} \Leftrightarrow \eta\mu x \cdot \sigma\upsilon\nu x = \frac{3\sqrt{2}}{12} \Leftrightarrow 6\eta\mu x \cdot \sigma\upsilon\nu x = \frac{3\sqrt{2}}{2} \Leftrightarrow$$
$$\Leftrightarrow 3\eta\mu 2x = \frac{3\sqrt{2}}{2} \Leftrightarrow \eta\mu 2x = \frac{\sqrt{2}}{2} \Leftrightarrow \eta\mu 2x = \eta\mu \frac{\pi}{4}$$

$$2x = \frac{\pi}{4} \quad \text{ή} \quad 2x = \pi - \frac{\pi}{4}$$

$$x = \frac{\pi}{8} \quad \text{ή} \quad x = \frac{3\pi}{8}$$

ΘΕΜΑ Γ

Γ1. Το $Q(x) = x^2 - x - 2$ είναι παράγοντας.

Επομένως

$$\left. \begin{array}{l} P(-1) = 0 \\ P(2) = 0 \end{array} \right\} \begin{array}{l} -\alpha - 1 + 5 - \beta + 1 = 0 \\ 8\alpha - 4 - 10 - \beta + 1 = 0 \end{array} \left\} \begin{array}{l} \alpha + \beta = 5 \\ 8\alpha - \beta = 13 \end{array} \right\} \oplus \begin{array}{l} \alpha = 2 \\ \beta = 3 \end{array}$$

Γ2.

$$\begin{array}{r} 2x^3 - x^2 - 5x - 2 \quad | -x^2 + 1 \\ -2x^3 \quad + 2x \quad \quad | -2x + 1 \\ \hline -x^2 - 3x - 2 \quad \quad \quad | \\ x^2 \quad - 1 \quad \quad \quad | \\ \hline -3x - 3 \end{array}$$

$$P(x) = (-x^2 + 1)(-2x - 1) + (-3x - 3).$$

Γ3.

$$P(x) + 3(x+1) \leq 0 \Leftrightarrow$$

$$\Leftrightarrow (-x^2 + 1)(-2x - 1) + (-3x - 3) + 3(x+1) \leq 0 \Leftrightarrow$$

$$\Leftrightarrow (-x^2 + 1)(-2x - 1) \leq 0$$



	$-\infty$	-1	$1/2$	1	$+\infty$		
$-x^2+1$	-	○	+	+	○	-	
$-2x+1$	+		+	○	-	-	
	-	○	+	○	-	○	+

$$x \in (-\infty, -1] \cup \left[\frac{1}{2}, 1\right].$$

ΘΕΜΑ Δ

Δ1.

$$\left. \begin{array}{l} \alpha + 2\beta - \gamma = 5 \\ 2\alpha - 5\beta + 3\gamma = 7 \\ 3\alpha + \beta - 4\gamma = 2 \end{array} \right\} \gamma = \alpha + 2\beta - 3$$

$$\left. \begin{array}{l} 2\alpha - 5\beta + 3(\alpha + 2\beta - 3) = 7 \\ 3\alpha + \beta - 4\alpha - 8\beta + 12 = 2 \end{array} \right\} \begin{array}{l} 5\alpha + \beta = 16 \\ -\alpha - 7\beta = -10 \end{array} \Leftrightarrow \begin{array}{l} 35\alpha + 7\beta = 112 \\ -\alpha - 7\beta = -10 \end{array} \oplus \alpha = 3, \beta = 1.$$

Δ2.

$$P(x) = 3x^3 + x^2 + 2x - 6$$

$$P(x) = 2 \Leftrightarrow 3x^3 + x^2 + 2x - 6 = 0$$

$$\begin{array}{r|rrrr} 3 & 1 & 2 & 6 & 1 \\ & & 3 & 4 & 6 \\ \hline & 3 & 4 & 6 & 0 \end{array}$$

$$(x-1)(3x^2 + 4x + 6) = 0 \Leftrightarrow x = 1.$$

Διότι, $3x^2 + 4x + 6 = 0$ είναι αδύνατη, επειδή $\Delta < 0$.

Δ3.

$$\frac{3x^3 + x^2 + 2x - 6}{6x^2 + 8x + 12} \leq \frac{x^2 - 4x + 3}{2} \Leftrightarrow$$

$$\Leftrightarrow \frac{(x-1) \cdot (3x^2 + 4x + 6)}{2(3x^2 + 4x + 6)} \leq \frac{x^2 - 4x + 3}{2} \quad \begin{matrix} 3x^2 + 4x + 6 > 0 \\ \Leftrightarrow \end{matrix} \frac{x-1}{2} \leq \frac{x^2 - 4x + 3}{2} \Leftrightarrow$$

$$x^2 - 4x + 3 - x + 1 \geq 0 \Leftrightarrow x^2 - 5x + 4 \geq 0 \Leftrightarrow (x-1)(x-4) \geq 0$$

	$-\infty$		1		4		$+\infty$
$x-1$	-		○	+		○	+
$x-4$	-			-		○	+
		+	○		-	○	+

$$x \in (-\infty, 1] \cup [4, +\infty).$$

Δ4. $\sqrt{x^2 - 5x + 4} = \sqrt{x - 5}$

Περιορισμοί:

- $x^2 - 5x + 4 \geq 0 \Leftrightarrow x \in (-\infty, 1] \cup [4, +\infty)$
- $x - 5 \geq 0 \Leftrightarrow x \in [5, +\infty)$

Συναλήθευση: Η εξίσωση θα λυθεί στο $[5, +\infty)$.

Υψώνω την εξίσωση στο τετράγωνο:

$$x^2 - 5x + 4 = x - 5$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3 \quad (\text{ΑΔΥΝΑΤΗ})$$

διότι $x \in [5, +\infty)$